Anoka-Hennepin Secondary Curriculum Unit Plan

Department:	**MATH**	**Course:**	**Advanced Algebra (H)**	**Unit 11 Title:**	**Polynomials and Rational Functions**	**Grade Level(s):**	**11**
Assessed Trimester:	**Trimester C**	**Pacing:**	**21-22 Days**	**Date Created:**	**6/25/2014**	**Last Revision Date:**	**6/25/2014**

Course Understandings: *Student will understand that:*

A. Relationships exist between real-world situations, mathematical equations, and graphs for sequences, series, polynomial functions, and exponential functions.
B. Sequences, series, polynomial, and exponential function can be categorized by form and that each form has specific processes to consider when solving and graphing.
C. There are a variety of strategies of varying efficiency for simplifying sequences, series, polynomial, and exponential expressions.
D. The numeric elements of a function have specific transformational effects on the graphs of those functions.
E. The context of a problem is important in recognizing the reasonableness of a solution.
F. There are benefits and limitations in the use of calculators and other technology to solve mathematical situations.

DESIRED RESULTS (Stage 1) - WHAT WE WANT STUDENT TO KNOW AND BE ABLE TO DO?

Established Goals

Minnesota State/Local/Technology Standard(s) addressed (2007):

- **Standard (9.2.1.#):** Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.
  Benchmark:
  - **9.2.1.1** Understand the definition of a function. Use functional notation and evaluate a function at a given point in its domain.
  - **9.2.1.2** Distinguish between functions and other relations defined symbolically, graphically or in tabular form.
  - **9.2.1.3** Find the domain of a function defined symbolically, graphically or in a real-world context.
  - **9.2.1.4** Obtain information and draw conclusions from graphs of functions and other relations.
  - **9.2.1.6** Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.
  - **9.2.1.7** Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods.
  - **9.2.1.8** Make qualitative statements about the rate of change of a function, based on its graph or table of values.
  - **9.2.1.9** Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations.

- **Standard (9.2.2.#):** Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.
  Benchmark:
  - **9.2.2.6** Sketch the graphs of common non-linear functions such as $f(x)=\sqrt{x}, f(x)=|x|$, $f(x)=\frac{1}{x}$, $f(x)=x^3$, and translations of these functions, such as $f(x)=\sqrt{x-2}+4$. Know how to use graphing technology to graph these functions.

- **Standard (9.2.3.#):** Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.
  Benchmark:
  - **9.2.3.1** Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified points in their domains.
  - **9.2.3.4** Add, subtract, multiply, divide and simplify algebraic fractions.

- **Standard (9.2.4.#):** Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential and n^{th} root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.
  Benchmark:
  - **9.2.4.8** Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.

Transfer

Students will be able to independently use their learning to: (product, high order reasoning)
- Model, analyze and solve real world situations using rational functions.

Meaning

Unit Understanding(s):	**Essential Question(s):**
Students will understand that: ● A real-world situation can be represented as a rational function ● A reasonable solution exists to rational problems and how to find those solutions.	**Students will keep considering:** ● Where can I find situations involving rational functions in the real world? ● When looking at a rational function how do the significant features of the formulas relate to real world representations? ● How do I decide which formula or method to use to solve a rational function? ● How do the skills and knowledge that we are learning influence the task of understanding situations that can be modeled by rational functions?

Acquisition

Knowledge - Students will: ● Know if a solution is extraneous in a rational equation. ● Demonstrate understanding of the relationship between different forms of rational equations and their graphs. ● Understand the definition of a rational function and the vocabulary of the significant features of a graph (intercepts, asymptotes, holes, intervals of increasing/decreasing). ● Understand the definition of a domain and range (and the restrictions placed on these given the function) ● Factor polynomials ● Simplify fractions of monomials. ● Distinguish between a common factor and a common term (example: $\frac{x^2+5x+6}{x^2+3x+2}$) **Reasoning - Students will:** ● Demonstrate understanding of the significant features of a graph of a rational function and their relationship to real-world situations (asymptotes, symmetry, intercepts, domain and range) ● Understand certain functions have a restricted domain and range ● Draw qualitative conclusions based on the graphs ● Interpret rational functions to solve real world situations ● Identify an expression as polynomial, rational, radical, logarithmic and exponential ● Interpret a solution in the original context	**Skills - Students will:** ● Identify the significant features of a graph of a rational function (asymptotes, holes, symmetry, intercepts, domain and range,) ● Graph rational functions ○ With a graphing calculator ● Use algebraic methods, tables and graphs to solve rational equations including real world situations and translate between representations. ● Interpret a solution in the original context ● Compare solutions to appropriate graphical or numerical estimates ● Simplify rational expressions using addition, subtraction, multiplication and division ● Use a graphing calculator to justify the equivalency of two expressions ● Solve rational inequalities graphically and algebraically

Common Misunderstandings

- Students forget to get a common denominator when adding or subtracting fractions
- Reducing within 'terms' rather than factors
- Students forget factoring patterns
- Students get confused between how to "simplify" vs. "multiply/divide" vs. "add/subtract" vs. "solve" because of the overlap of techniques used in different ways
- Students will incorrectly add terms that are not like terms (i.e. $2a+3b = 5ab$ or $2x^2 + 5x = 7x^3$).
- Students incorrectly subtract polynomial expressions as shown below:

  $(3x^2-5x+7)-(x^2-3x-2)$ vs. $3x^2-5x+7-x^2-3x-2$ (not an equivalent expression)

- Students will neglect partial products when multiplying polynomials. For example, some students incorrectly simplify the expression $(x-3)^2$ by writing $(x)^2+(-3)^2$ and ending up with x^2+9 for an answer. Some students incorrectly apply a memorized "FOIL" procedure when simplifying expressions like $(x^2-3x+9)(x+2)$.
- Students will incorrectly add algebraic fractions (e.g. $\frac{3}{x-1} + \frac{2}{1-x} = \frac{5}{x-1}$) because they fail to realize that subtraction is not commutative (i.e. $x-1$ is not equivalent to $1-x$).
- Students will incorrectly "cancel out" terms instead of factors when simplifying algebraic fractions (i.e. $\frac{x-5}{x+2} = \frac{-5}{2}$.
- A common strategy to correctly solve equations involving algebraic fractions (rational equations) is to multiply both sides of the equation by the least common denominator to correctly generate an equivalent equation that have denominators of one and is easier for students to solve symbolically.

$$\frac{20}{x+2} + \frac{15}{x+3} = 8$$

$$(x+2)(x+3)\frac{20}{x+2} + (x+2)(x+3)\frac{15}{x+3} = (x+2)(x+3)8$$

$$(x+3)20+(x+2)15=(x+2)(x+3)8$$

Some students incorrectly apply this same "clearing the fraction" strategy when working with expressions.

1) $\frac{20}{x+2} + \frac{15}{x+3}$

2) $(x+2)(x+3)\frac{20}{x+2} + (x+2)(x+3)\frac{15}{x+3}$ Expression in (2) is not equivalent to expression in (1)

3) $(x+3)20+(x+2)15$

Students incorrectly subtract fractions by forgetting to subtract the entire expression in the numerator of the fraction on the right of the subtraction symbol.

1) $\frac{5x+10}{(x+2)(x+3)} - \frac{3x+6}{(x+2)(x+3)}$

2) $\frac{5x+10-3x+6}{(x+2)(x+3)}$

   Expression in (2) is not equivalent to expression in (1). Student did not subtract the entire numerator from the fraction on the right of the equal sign.

3) 2x+16(x+2)(x+3)

   This expression is the student's final answer and is not equivalent to the expression in (1) but is equivalent to the expression in (2).

Essential new vocabulary

- Rational numbers
- Hyperbola
- Complex fraction
- Hole